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Proceedings of the 5th International Conference on Inverse Problems in Engineering: Theory and Practice, Cambridge, UK, 11-15th July 2005

2D AND 3D INVERSE BOUNDARY AND INVERSE GEOMETRY BEM SOLUTION IN CONTINUOUS CASTING

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Abstract - This paper discusses an inverse solution consisting of the boundary condition estimation and the phase change front identification in continuous casting process. The solution procedure utilizes sensitivity coefficients and temperature measurements inside the solid phase. The algorithms proposed make use of the Boundary Element Method (BEM) in both 2D and 3D case. With the purpose of limiting a number of estimated values (and consequently the number of temperature sensors) the Bezier splines (in 2D case) and the Bezier surfaces (in 3D case) are employed for modelling the interface between the solid and liquid phases. For the same reason heat flux distribution along the cooling boundary is approximated using spline function or broken line. In order to demonstrate the main advantages of the developed formulation, continuous casting of copper was considered as a numerical example.

Keywords - inverse geometry problem, inverse boundary problem, sensitivity coefficients, BEM solution of continuous casting, Bezier splines, Bezier surfaces.

1. INTRODUCTION

Growing demand for high-quality alloys possessing specific properties stimulates frequent application of the continuous casting process in the metallurgical industry. In order to be able to guarantee the required quality of the casted metal, the whole casting process must be carefully controlled, including proper designing of the cooling system. This is the reason why an accurate determination of the location of the interface between liquid and solid phases were considered in this work. The proposed numerical procedures are based on the sensitivity analysis and boundary element method [6, 1, 9, 12].

The solidification of metal or alloy takes place in a mould (crystallizer) cooled by a flowing water. The liquid material flows into the mould having the walls cooled by flowing water. The solidifying ingot is pulled out of the mould by withdrawal rolls. It is also very intensively cooled outside the crystallizer (by water sprayed over the surface).

These problems were the topics of works dealing with the boundary and the geometry inverse problems formulated as 2D ones [11, 7, 13, 10, 8]. The boundary inverse problem consisted of the determination of the heat flux distribution along outer boundary of the ingot. The numerical procedures and the results obtained were presented in [11]. The geometry inverse problem concerned the estimation of the location of the phase change front. The publications [8, 7] discuss the details of the solution algorithms and method of modelling the interface shape. The influence of the number and accuracy of measurements were also investigated. Particularly, attention was paid to the application of the Bezier splines for the phase change front approximation [10] and using sensitivity analysis leading to the determination of the sensitivity coefficients (in case of the quadratic or cubic boundary elements) [8, 10].

The same approach is continuously discussed in the presented paper. Bezier surfaces being 3D generalization of 2-dimensional Bezier splines are applied in both inverse boundary and/or inverse geometry problems. The Bezier splines or surfaces approximate the real phase change front whose shape and location are controlled by coordinates of the Bezier control points.

It has to be stressed that the mathematical model of the inverse boundary problem as well as inverse geometry problem include the solid ingot only. It means that the problem is solved for the solid phase and its interaction with the liquid phase manifests itself in the collected temperature measurements. Taking all these into account this paper should be seen as a natural extension of the developed algorithms for the 3D cases for both boundary and geometrical inverse problems.

2. PROBLEM FORMULATION

A brief description of the mathematical model of the direct heat transfer problem for the continuous casting process is discussed in the first part of this section. This model serves as a basis for the both inverse boundary and inverse geometry problems.

The mathematical description of the considered phenomena, defined as the 3D steady-state diffusion-convection heat transfer, consists of:

- a convection-diffusion equation for the solid ingot:

$$\nabla^2 T(\mathbf{r}) - \frac{1}{a} v_x \frac{\partial T}{\partial x} = 0 \tag{1}$$

  where $T(\mathbf{r})$ is the temperature at point $\mathbf{r}$, v_x stands for the casting velocity (assumed to be constant and having only one component along the x-direction) and a is the thermal diffusivity of the solid phase.

- boundary conditions defining the heat transfer process along the boundaries (Figure 1), including the specification of the melting temperature along the phase change front:

$$T(\mathbf{r}) = T_m, \qquad \mathbf{r} \in \Gamma_{ABCD} \tag{2}$$

$$-\lambda \frac{\partial T}{\partial n} = q(\mathbf{r}), \qquad \mathbf{r} \in \Gamma_{ABFE} \cup \Gamma_{BCGF} \tag{3}$$

$$T(\mathbf{r}) = T_s, \qquad \mathbf{r} \in \Gamma_{EFGH} \tag{4}$$

$$-\lambda \frac{\partial T}{\partial n} = 0, \qquad \mathbf{r} \in \Gamma_{ADHE} \cup \Gamma_{DCGH} \tag{5}$$

  where T_m stands for the melting temperature, T_s is the ingot temperature while leaving the system, λ is the thermal conductivity and q is the heat flux. Both symbols T_m and T_s represent constant temperatures.

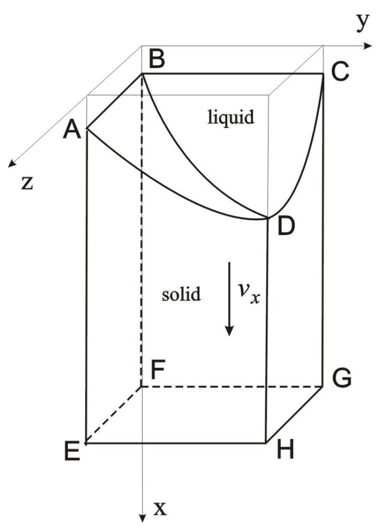

Figure 1: Scheme of the 3D domain of continuous casting system .

In the inverse analysis, some information within the mathematical model of the direct heat transfer problem is unknown or uncertain. This means that such mathematical description needs to be supplemented by the measurements. In the majority of considered examples these measurement data are numerically simulated, although it is also possible to use measurements obtained through experiment, *e.g.* [13].

In the inverse heat transfer problem for continuous casting the location of the phase change surface Γ_{ABCD} (where the temperature is equal to the melting one) or the heat flux along surfaces Γ_{ABFE} and Γ_{BCGH} is unknown. In order to complete the mathematical model, it is assumed that the temperatures U_i were measured inside the ingot using the technique known as L-rod technique [3, 4]. These measurements are then collected in a vector $\mathbf{U}$.

Depending on the type of the inverse problem (boundary or geometry one), the objective is to estimate the identified values uniquely describing the location of the phase change front or the heat flux distribution along the side surfaces. These identified values are collected in the vector $\mathbf{Y} = [y_1, \ldots, y_n]^T$.

In the boundary inverse problems components of vector $\mathbf{Y}$ are connected to the heat flux distribution. In case of the geometry inverse problem this vector contains components defining the location of the phase change front. In both cases they are the coordinates of the Bezier control points. To provide a distinction of the two kinds of inverse problem, components y_j can be denoted by q_j for the inverse boundary or v_j for the inverse geometry problem. The ways of modelling both quantities are discussed in the subsequent section.

Because of the ill-posed nature of all inverse problems, the number of measurements should be appropriate to make the problem overdetermined. This is achieved by using a number of measurement points greater than the number of design variables. Thus, in general, inverse analysis leads to the optimization procedures with least squares calculations of the objective functions Δ. However, in the cases studied here, an additional term needed to improve stability was also introduced [9, 6], *i.e.*

$$\begin{aligned} \Delta \;=\; & (\mathbf{T}_{cal} - \mathbf{U})^T \, \mathbf{W}^{-1} \, (\mathbf{T}_{cal} - \mathbf{U}) \, + \\ & \left(\mathbf{Y} - \tilde{\mathbf{Y}}\right)^T \mathbf{W}_Y^{-1} \left(\mathbf{Y} - \tilde{\mathbf{Y}}\right) \rightarrow \min \end{aligned} \tag{6}$$

where vector $\mathbf{T}_{cal}$ contains temperatures calculated at the sensor locations, $\mathbf{U}$ stands for the vector of temperature measurements and superscript T denotes transpose matrices. The symbol $\mathbf{W}$ denotes the covariance matrix of measurements. Thus, the contribution of more accurately measured data is stronger than the data obtained with lower accuracy. Known prior estimates of design vector components are collected in vector $\tilde{\mathbf{Y}}$, and $\mathbf{W}_Y$ stands for the covariance matrix of prior estimates. The coefficients of the matrix $\mathbf{W}_Y$ have to be large enough to catch the minimum (these coefficients tend to infinity if prior estimates are not known). It was found that the additional term in the objective function place similar role to the regularisation term in other approaches. Containing prior estimates, this term is very important in the inverse analysis, since it considerably improves the stability and accuracy of the inverse procedure.

Generally, the inverse problem is solved by building up a series of direct solutions which gradually approach the correct values of design variables. This procedure can be expressed by the following main steps:

1. make the boundary problem well-posed. This means that the mathematical description of the thermal process is completed by assuming arbitrary but known values $\mathbf{Y}^*$ (as required by the direct problem).
2. solve the direct problem obtained above and calculate temperatures $\mathbf{T}^*$ at the sensor locations; compare these temperatures and measured values $\mathbf{U}$ and modify the assumed data y_j^*, $j = 1, 2, \ldots, n$; if the inverse problem is non-linear (i.e. geometrical one) this point should be repeated until v_j converges [8, 7, 10].

It is also possible to define an inverse problem as a combination of both boundary and geometry one. In such an approach the above algorithm has to be expanded. Step 2. is now split into boundary and geometry substeps in which values connected with the first substep are estimated keeping values connected with the second one unchanged. Details of such methodology were presented in [13].

For all kinds of inverse problems, the above algorithm applies the sensitivity analysis and the minimization of the objective function (6) leading to the following set of equations [9, 8]:

$$\begin{aligned} \left(\mathbf{Z}^T \, \mathbf{W}^{-1} \, \mathbf{Z} + \mathbf{W}_{\mathbf{Y}}^{-1}\right) \mathbf{Y} \;=\; & \mathbf{Z}^T \, \mathbf{W}^{-1} \left(\mathbf{U} - \mathbf{T}^*\right) + \\ & \left(\mathbf{Z}^T \, \mathbf{W}^{-1} \, \mathbf{Z}\right) \mathbf{Y}^* + \mathbf{W}_{\mathbf{Y}}^{-1} \, \tilde{\mathbf{Y}} \end{aligned} \tag{7}$$

where the sensitivity coefficients (determined at the measurement points) are collected in the matrix $\mathbf{Z}$.

The sensitivity coefficients, as the main concept of the sensitivity analysis are the derivatives of the temperature at point i with respect to the identified value at point j, *i.e.*

$$Z_{ij} = \frac{\partial T_i}{\partial y_j} \tag{8}$$

The sensitivity coefficients provide a measure of each identified value and indicate how much it should be modified due to change of temperature differences.

The sensitivity coefficients are obtained by solving a set of auxiliary direct problems in succession. Each of these direct problems arise through differentiation of equation (1) and corresponding boundary conditions (2)−(5) with respect to the particular component of the vector $\mathbf{Y}$. For the sake of different nature of both kinds of problems, it is necessary to build and solve different direct problems connected with y_j values.

In this work, the BEM [1, 12] is applied for solving both the direct thermal and the sensitivity coefficient problem. The main advantage of using this method is the simplification in meshing since only the boundary has to be discretized. This is particularly important in the inverse geometry problems in which the geometry of the body is modified within each iteration step. Furthermore, the location of the internal measurement sensors does not affect the discretization. Finally, in the heat transfer analysis, the BEM solution directly provides temperatures and heat fluxes, which are both required by the inverse solution. In other words, the numerical differentiation of temperature (*i.e.*, numerical calculations of the heat fluxes) is not needed.

The BEM system of equations has boundary-only form for both the thermal and the sensitivity coefficient problems

$$\mathbf{H}\,\mathbf{T} = \mathbf{G}\,\mathbf{Q} \tag{9}$$

$$\mathbf{H}\,\mathbf{Z} = \mathbf{G}\,\mathbf{Q_Z} \tag{10}$$

where $\mathbf{H}$ and $\mathbf{G}$ stand for the BEM influence matrices. Depending on the dimensionality of the problem, the fundamental solution to the convection-diffusion equation is expressed by the following formulae [1, 12]

$$u^* = \begin{cases} \dfrac{1}{2\pi\lambda} \exp\left(-\dfrac{v_x\, r_x}{2a}\right) \mathrm{K}_0\left(\dfrac{|v_x|\, r}{2a}\right) & \text{2–D} \\[2ex] \dfrac{1}{4\pi r\lambda} \exp\left[\dfrac{v_x\,(r - r_x)}{2a}\right] & \text{3–D} \end{cases} \tag{11}$$

where K_0 stands for the Bessel function of the second kind and zero order, r is the distance between source and field points, with its component along the x-axis denoted by r_x.

3. DETERMINATION OF IDENTIFIED VALUES

As mentioned before, the ill-conditioned nature of all inverse problems requires that they have to be made overdetermined. On the other hand, it is very important to limit the number of sensors, mainly because of practical difficulties connected with measurement acquisition. Application of Bezier splines or surfaces allows the modelling of the phase change front using a considerably smaller number of design variables as well as the approximation the heat flux distribution by broken line or some spline functions.

Application of Bezier splines/surfaces - Application of the Bezier splines/surfaces allows to define a location of the phase change front together with the limitation of the number of identified values.

In 2D problem the Bezier curves are applied. They are made up of the cubic segments based on four control points $\mathbf{V}_0$, $\mathbf{V}_1$, $\mathbf{V}_2$ and $\mathbf{V}_3$. The following formula presents the definition of the cubic Bezier segments:

$$\begin{aligned} \mathbf{P}(u) &= (1-u)^3\,\mathbf{V}_0 + 3\,(1-u)^2\,u\,\mathbf{V}_1 + \\ &\quad 3\,(1-u)\,u^2\,\mathbf{V}_2 + u^3\,\mathbf{V}_3 \end{aligned} \tag{12}$$

where $\mathbf{P}(u)$ stands for any point on the Bezier curve, and u varies in the range [0,1], [5].

Numerical experiments have shown that the Bezier curve composed of two cubic segments satisfactorily approximates the phase change front [2]. Apart from limitations of the identified values, the application of the Bezier curves (cubic polynomials) has to ensure smoothness of the boundary. The collinear location of appropriate control points makes the whole boundary smooth even at points which are shared by neighbouring segments [10].

The vector $\mathbf{Y} = [y_1, \ldots, y_{2n}]^T$ can be written as $\mathbf{Y} = [v_1^x, v_1^y \ldots, v_n^x, v_n^y]^T$ where v_i^x, v_i^y are the x and y coordinates of the given control point. Actually, some of these coordinates are defined by additional conditions resulting from the physical nature of the problem. In consequence, the number of identified values can be limited to ten [7, 10], which also means a reduction of the number of required measurements.

In 3D approach, a location of the phase change front is described by the Bezier surface. This is a 3D equivalent of the Bezier splines applied in 2D problems.

Again the shape and location of the Bezier surface is defined by the control points. In this case there are 16 points $\mathbf{V}_{00}$, $\mathbf{V}_{01}$, $\mathbf{V}_{02}$, $\mathbf{V}_{03}$, $\mathbf{V}_{10}$, $\mathbf{V}_{11}$, $\mathbf{V}_{12}$, $\mathbf{V}_{13}$, $\mathbf{V}_{20}$, $\mathbf{V}_{21}$, $\mathbf{V}_{22}$, $\mathbf{V}_{23}$, $\mathbf{V}_{30}$, $\mathbf{V}_{31}$, $\mathbf{V}_{32}$, $\mathbf{V}_{33}$ and the following formulae present the definition of the Bezier surface:

$$\mathbf{P}(u,v) = \begin{bmatrix} \mathbf{V}_{00} & \mathbf{V}_{01} & \mathbf{V}_{02} & \mathbf{V}_{03} \\ \mathbf{V}_{10} & \mathbf{V}_{11} & \mathbf{V}_{12} & \mathbf{V}_{13} \\ \mathbf{V}_{20} & \mathbf{V}_{21} & \mathbf{V}_{22} & \mathbf{V}_{23} \\ \mathbf{V}_{30} & \mathbf{V}_{31} & \mathbf{V}_{32} & \mathbf{V}_{33} \end{bmatrix} \cdot \begin{bmatrix} (1-u)^3 \\ 3\,(1-u)^2\; u \\ 3\,(1-u)\; u^2 \\ u^3 \end{bmatrix} \cdot \begin{bmatrix} (1-v)^3 \\ 3\,(1-v)^2\; v \\ 3\,(1-v)\; v^2 \\ v^3 \end{bmatrix}^T \tag{13}$$

where $\mathbf{P}(u,v)$ stands for any point on the Bezier surface, u and v both vary in the range [0,1] while symbol $\cdot$ means the product of matrices.

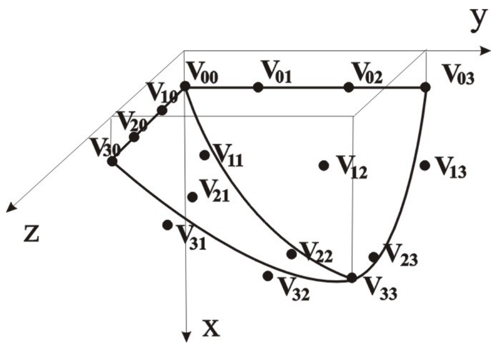

Figure 2: Scheme of Bezier surface with control points.

Comparing 3D and 2D cases the number of values having an effect on the Bezier surface shape in 3D problem is fairly big($16 \cdot 3$ values). In reality, like in 2D formulation, some of these values (coordinates of Bezier control points) are defined by additional constraints resulting from the physical nature of the problem. For example, the y and z-coordinates of the control points located on the ingot side surfaces and on the symmetry surfaces are known, because the dimensions of the ingot are fully determined. Moreover, the equality of the x-coordinate of appropriate control points ensures the existence of derivatives on the symmetry surfaces.

In this paper, it was additionally assumed, that only x coordinate of the Bezier control points is estimated. In fact, those values have main influence on the Bezier surface shape (Figure 2). All the assumptions cause that the phase change front can be described by the following formula:

$$x(u,v) = \begin{bmatrix} x_{00} & x_{01} & x_{02} & x_{03} \\ x_{10} & x_{11} & x_{12} & x_{13} \\ x_{20} & x_{21} & x_{22} & x_{23} \\ x_{30} & x_{31} & x_{32} & x_{33} \end{bmatrix} \cdot \begin{bmatrix} (1-u)^3 \\ 3\,(1-u)^2\; u \\ 3\,(1-u)\; u^2 \\ u^3 \end{bmatrix} \cdot \begin{bmatrix} (1-v)^3 \\ 3\,(1-v)^2\; v \\ 3\,(1-v)\; v^2 \\ v^3 \end{bmatrix}^T \tag{14}$$

where x_{ij} is the first coordinate of Bezier control points, $x(u,v)$ stands for x-coordinate of points on Bezier surface and $y(u,v)$ and $z(u,v)$ are calculated analogously to (13). It should also be noted that in reality the location of phase change front depends on five values only, *i.e.* x_{00}, x_{11}, x_{12}, x_{21} and x_{22}.

4. SENSITIVITY COEFFICIENTS

The calculation of the sensitivity coefficients requires solution of the set of direct problems similar to the thermal one. Each of these problems arise through differentiation of equation (1) and corresponding boundary conditions (2)-(5) with respect to the particular design variable y_j. As mentioned before, depending on the type of the considered problem, the design variable describes the boundary conditions along the side surface of the ingot or the location of the phase change front. In both cases the resulting field Z_j is governed by an equation having the form:

$$\nabla^2 Z_j(\mathbf{r}) - \frac{1}{a}\, v_x\, \frac{\partial Z_j}{\partial x} = 0 \tag{15}$$

where $Z_j = \dfrac{\partial T}{\partial y_j}$ and $y_j = q_j$ in the inverse boundary problem or $y_j = v_j$ in the inverse geometry problem.

The inverse boundary problem - In the inverse boundary problem in which the boundary conditions along Γ_{ABFE} and Γ_{BCGF} are identified, the heat flux distribution has to be found.

Cooling conditions of the continuous casting process permit to assume that the heat flux distribution varies mainly in the direction of pulling of the ingot (in direction orthogonal to the axis of symmetry the heat flux is constant). It means that the heat flux distribution can be approximated by appropriate spline function or broken line, based on n parameters which can be denoted as $q(\mathbf{r}) = f(q_1, \ldots, q_n)$. The number of these parameters does not affect the way of calculations and cause only increasing of the number of the identified values. In the continuous casting of copper [9, 6] the heat flux varies linearly along the mould and exponentially along the water spray. It means that the heat flux distribution is described only by three values. In some other cases, *e.g.* in the continuous casting of alloy of aluminum, it is better to approximate the heat flux distribution by broken line [13]. The heat fluxes q_i are components of vector $\mathbf{Y}$ whose estimation is an objective of the problem.

In the inverse boundary problem the sensitivity coefficients are calculated from the auxiliary direct problem containing the governing equation (15) and homogenous boundary conditions along boundaries Γ_{ABCD}, Γ_{DCGH}, Γ_{ADHE} and Γ_{EFGH}. Condition along the surfaces Γ_{ABFE} and Γ_{BCGF} is nonhomogeneous and the whole system reads as:

$$Z_j(\mathbf{r}) = 0, \qquad \mathbf{r} \in \Gamma_{ABCD} \tag{16}$$

$$-\lambda \frac{\partial Z_j}{\partial n} = \frac{\partial f(q_1, q_2, \ldots, q_n)}{\partial q_j} \qquad \mathbf{r} \in \Gamma_{ABFE} \cup \Gamma_{BCGF} \tag{17}$$

$$Z_j(\mathbf{r}) = 0, \qquad \mathbf{r} \in \Gamma_{EFGH} \tag{18}$$

$$-\lambda \frac{\partial Z_j}{\partial n} = 0, \qquad \mathbf{r} \in \Gamma_{ADHE} \cup \Gamma_{DCGH} \tag{19}$$

where function f is an approximation of the heat flux distribution.

The calculated sensitivity coefficients are collected in the matrix $\mathbf{Z}$ and introduced into the system of equations (7). It has to be noted that the inverse boundary problem is a linear one which allows to solve it in non-iterative way.

The inverse geometry problem - In inverse geometrical problems, likewise in the boundary one, differentiation of the boundary conditions (2)-(5) produces conditions for the sensitivity coefficient direct problem. These conditions are of the same type as in the original thermal problem, but homogeneous. Of course in this case the original thermal problem is differentiated with respect to the values v_j determining the location of the phase change front. Particularly, the boundary condition along the front Γ_{ABCD} is obtained by differentiating equation (2) with respect to identified value v_j. Because along this boundary constant temperature T_m is expected, after differentiation the following equation is obtained:

$$\frac{\partial T}{\partial v_j} + \frac{\partial T}{\partial x} \cdot \frac{\partial x}{\partial v_j} + \frac{\partial T}{\partial y} \cdot \frac{\partial y}{\partial v_j} + \frac{\partial T}{\partial z} \cdot \frac{\partial z}{\partial v_j} = 0 \tag{20}$$

According to the definition of the sensitivity coefficient $Z_j = \dfrac{\partial T}{\partial v_j}$ the following equation is obtained:

$$Z_j = \frac{\partial T}{\partial v_j} = -\left(\frac{\partial T}{\partial x} \cdot \frac{\partial x}{\partial v_j} + \frac{\partial T}{\partial y} \cdot \frac{\partial y}{\partial v_j} + \frac{\partial T}{\partial z} \cdot \frac{\partial z}{\partial v_j} \right) \tag{21}$$

Using vector representation of the Fourier Law, (21) can be rewritten as:

$$Z_j = \frac{\partial T}{\partial v_j} = \frac{1}{\lambda} \left(q_x \cdot \frac{\partial x}{\partial v_j} + q_y \cdot \frac{\partial y}{\partial v_j} + q_z \cdot \frac{\partial z}{\partial v_j} \right) \tag{22}$$

where q_x, q_y and q_z are components of the heat flux vector in the global coordinate system. It is important to note that in heat transfer analysis, BEM solutions directly provide temperatures and heat fluxes at the nodes (in local coordinate system connected with boundary element). Making use of these BEM solutions the heat fluxes are rewritten into the global coordinate system. Thus, the boundary condition along the phase change surface has the form:

$$\frac{\partial T}{\partial v_j} = \frac{q_n}{\lambda}\left(\frac{n_x}{\|\mathbf{n}\|}\cdot\frac{\partial x}{\partial v_j} + \frac{n_y}{\|\mathbf{n}\|}\cdot\frac{\partial y}{\partial v_j} + \frac{n_z}{\|\mathbf{n}\|}\cdot\frac{\partial z}{\partial v_j}\right) \tag{23}$$

where $\mathbf{n} = [n_x, n_y, n_z]$ is the normal vector to the phase change surface and $\|\mathbf{n}\|$ means the length of $\mathbf{n}$.

The derivatives of x y and z with respect to the design variable v_j depend on the particular geometrical representation of the phase change front. In presented calculations they are obtained by differentiation of the formula (11) with respect to the design variable v_j. As it was mentioned, in the presented work only x-coordinates of the chosen Bezier control points are estimated. It means that partial derivatives of y and z vanish

$$\frac{\partial y}{\partial v_j} = \frac{\partial z}{\partial v_j} = 0. \tag{24}$$

Finally (23) reduces into

$$Z_i = \frac{\partial y}{\partial v_j} = \frac{\partial T}{\partial y_i} = \frac{q_n}{\lambda}\cdot\frac{n_x}{\|\mathbf{n}\|}\cdot\frac{\partial x}{\partial y_i} \tag{25}$$

The above derivation is valid in both 3D and 2D case (if need without parts connected with z coordinate).

It has to be noted that the inverse geometry problems are always non-linear. It means that the iteration procedure has to be applied and this procedure is conducted till the convergence criteria are satisfied.

5. NUMERICAL RESULTS

In the presented work 3D inverse boundary and inverse geometry problems were solved. The boundary conditions and the phase change front location were estimated by using measurements generated numerically. Previous tests showed that the best results were obtained if sensors were located as close as possible to the identified values. Thus, in the problems under consideration, the measurement points are arranged at a uniform rate under interface between solid and liquid part. The orthogonal projection onto yz-plane of the ingot and the thermocouples position (each of them collects five measurements) is presented in Fig.3.

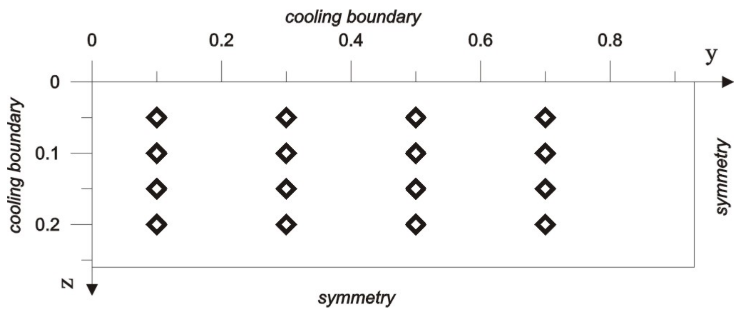

Figure 3: The orthogonal projection of ingot and sensor points onto yz-plane.

A majority of previous studies were devoted to boundary, geometry and combined boundary-geometry inverse problems formulated as 2-dimensional ones [8, 11, 10, 7, 13]. In order to test the algorithm verification, results obtained in 2D boundary-geometry inverse problem were additionally compared to the results obtained by others authors [3]. Gained experience permits us to adapt the developed method to 3D case. In order to show the main advantages of the proposed algorithms the continuous casting problem from the copper industry was taken under consideration.

In these calculations it is assumed that the heat fluxes vary linearly along the mould and exponentially along the water-sprayed boundary [11]. The following heat fluxes $q_1 = 1\cdot10^6$ W/m^2, $q_2 = 5\cdot10^4$ W/m^2 and $q_3 = 1.5\cdot10^6$ W/m^2 were accepted. All results were obtained for the melting temperature $T_m = 1083$°C whereas the end temperature T_s was assumed to be 50°C. The phase change front has been modelled by the Bezier surface.

3D Inverse Boundary Problem - The temperature measurements inside the ingot were simulated numerically, adding random errors to the selected temperatures of reference field. Assumed measurement errors did not exceed 2%. The considerable influence of measurement accuracy on the quality of heat fluxes estimation were presented in previous works concerning 2D problems [11, 10]. Exact and estimated heat flux distributions obtained in 3D model are presented in Figure 4.

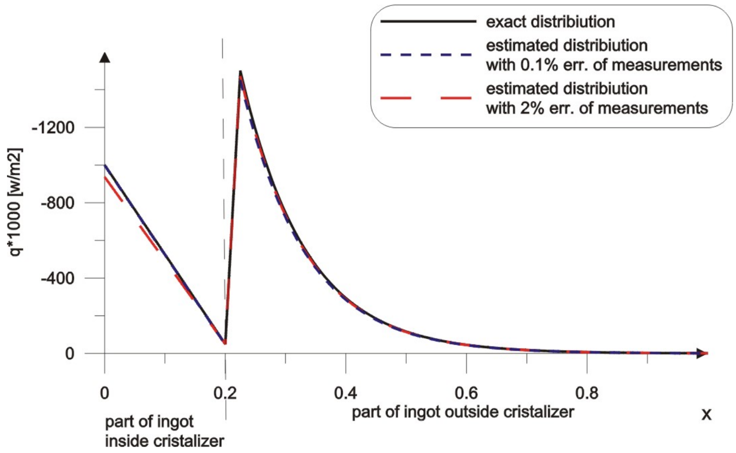

Figure 4: Comparison of exact and estimated heat flux distributions.

3D Inverse Geometry Problem - The considered problem is formulated like the one from previous subsection. Now, it is assumed that the governing equation (1) and all the boundary conditions are known. The location and shape of the phase change front need to be estimated. This front is modelled by one Bezier surface.

The temperature measurements inside the ingot were simulated numerically and perturbated by the random errors, which did not exceed 0.1%. Figure 5 presents the obtained agreement between measured and calculated temperatures at some sensor points (collected by 4 thermocouples) after an iteration procedure.

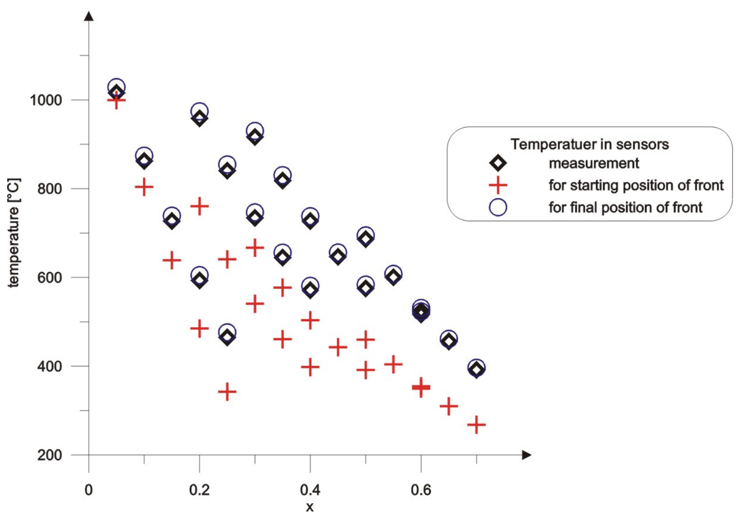

Figure 5: Measured and calculated temperatures at sensor points.

It has to be noted that differences between $\mathbf{T_{cal}}$ and $\mathbf{U}$ consist of the main part of the objective function (6). Thus this difference is a primary criteria of the obtained results assessment.

Because in presented problem the exact values of estimated quantities were known, it is also possible to compare the estimated and exact values of this quantities. Appropriate comparison is shown in the following table:

exact value	calculated value
0.02	0.0207
0.05	0.077
0.1	0.087
0.25	0.2552
0.4	0.4012

The position and shape of estimated solid-liquid determined determined by the estimated values found in the iteration procedure and presented in the table is shown in Figure 6.

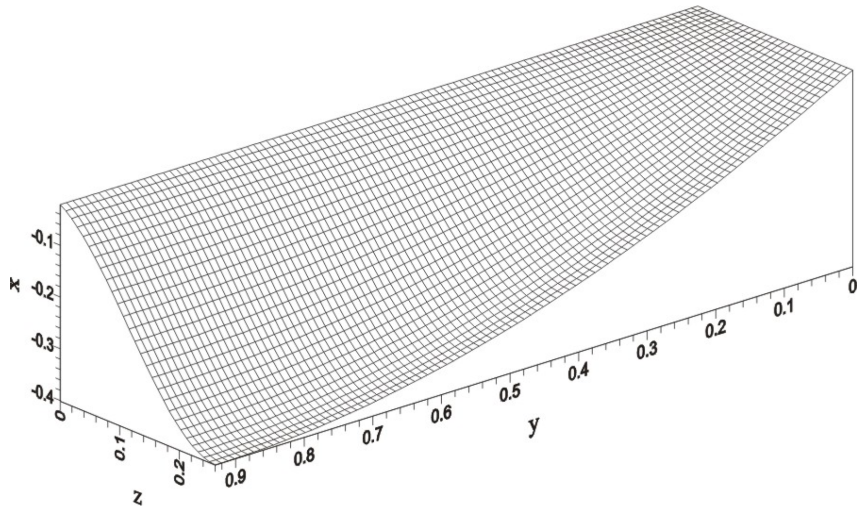

Figure 6: The phase change front location.

It has to be noted that on the contrary to the direct problems, in non-linear inverse problems, existence and uniqueness of solution is not obvious. Some of starting values may not fulfil the conditions allowing to solve the problem. Because of this, in discussed inverse geometry problem application of lumping procedure turned out to be necessary [7, 10]. This approach provides such a correction of initial front location that the process is convergent. The results presented above were obtained after application of this approach.

6. CONCLUSIONS

Results presented in the paper show that the extension of the geometry and the boundary inverse algorithms to the 3D case is possible. The proposed algorithm is a generalization of the method developed for 2D problems and does not require to be solved as a separate problem.

In order to limit the number of identified values, the phase change front was modelled by the Bezier surface and the heat flux distribution was approximated using the same concept.

The results obtained allow to think that the algorithms presented in the paper can be used in combined inverse boundary and geometry problem in 3D case and eventually in an industrial applications.

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